Приложение № 1

к распоряжению Финуниверситета

от «\_\_\_» \_\_\_\_\_\_\_\_2019 № \_\_\_\_\_\_\_

**Шаблон структуры и содержания рабочей программы дисциплины**

**Syllabus**

**1. Name of a subject – Mathematics**

The main learning outcome of this discipline is acquiring by a student the basic mathematical knowledge and skills (forming competences) for solving standard economical tasks.

**2. Mapping of learning outcomes (list of competences), with the relevant indicators described and subject learning outcomes indicated**

 The discipline provides necessary tools to form competences listed below.

The section lists the graduates’ coded competencies that are to be developed during the learning process, indicators that show their development (generalized descriptions of specific actions performed by the graduate that clarify and reveal the competence content), learning outcomes (knowledge, skills) with indicators of competence development (in the form of a table):

 Table 1

|  |  |  |  |
| --- | --- | --- | --- |
| Competence code | Competence  | Competence development indicators[[1]](#footnote-1) | Learning outcomes (skills and knowledge) and indicators that show competence development[[2]](#footnote-2)  |
| IK-4 | The ability to draw up analytical and reporting materials based on the results of the work performed. |  | Know the general rules for preparing analytical and reporting documents. Be able to draw up analytical and reporting materials logically, clearly and compactly.Possess the skills of using software packages for preparing analytical and reporting materials. |
| IK-5 | The ability to apply calculation techniques and basic research methods. |  | Know the basic concepts, calculation techniques and methods of linear algebra and mathematical analysis.Be able to solve typical mathematical problems, build and analyze mathematical models; apply mathematical methods to solve applied problems.Possess the skills of using mathematical tools to solve economic problems. |
| PKN-3 | Ability to apply mathematical methods to solve standard professional financial and economic problems, to interpret the obtained mathematical results. |  | Know the basic mathematical concepts and methods required to solve standard financial and economic problems.Be able to solve typical mathematical and financial and economic problems, interpret the results obtained.Possess the skills of mathematical modeling of economic phenomena and processes. |

**3. Place of the discipline subject in the curriculum**

The discipline *"Mathematics"* is a discipline of the Module of Mathematics and Computer Science of the direction of training 38.03.01 *"Economics"*.

The study of the discipline *"Mathematics"* is based on the knowledge gained in the framework of the school course of mathematics or the corresponding disciplines of secondary vocational education. The discipline "Mathematics" is the theoretical basis for all disciplines of the module of Mathematics and Computer Science, and mathematical concepts and methods are used in the future in the study of general professional disciplines and disciplines of the profile.

The section describes what place is occupied by the subject in the program curriculum.

**4. Workload in credits and academic hours, with class work (lectures and seminars) and self-study indicated**

The data are presented in the form of a table.

Table 2

|  |  |  |  |
| --- | --- | --- | --- |
| **Type of work**  | **Total** **(in credits and hours)**  | **Semester 1 (in hours)** | **Semester 2 (in hours)** |
| **Overall workload**  | ***6/216*** | ***108*** | ***108*** |
| ***Class work***  | ***108*** | ***36*** | ***36*** |
| *Lectures*  | ***36*** | ***18*** | ***18*** |
| *Seminars*  | ***72*** | ***36*** | ***36*** |
| ***Self-study***  | ***108*** | ***36*** | ***72*** |
| Formative assessment  |  | ***Test*** | ***Test*** |
| Summative assessment  |  | ***Exam*** | ***Full Exam*** |

**5. Subject content (with the thematic components indicated).**

**5.1. Thematic components content**

**Part 1. Calculus**

**Thematic component 1. Number sets and functions.**

Elements of set theory. Quantifiers. Operations on sets: union, intersection, difference, addition. Finite, countable and uncountable numbers. Limited and unlimited sets.

Sets of natural, integer, rational and real numbers. Complex numbers and operations with them. The modulus and argument of a complex number. Algebraic and trigonometric forms of notation of complex numbers. Function concept. Numeric function one standard. Methods for setting functions. Function graph. Even, odd, monotonic, periodic and limited functions. Concavity and inflection points.

Functions in economics: utility functions, one-factor production functions, supply and demand functions. Average cost functions and the relationship between them (ATC = AVC + AFC).

**Thematic component 2. Limits and continuity.**

Numerical sequences, limit of a sequence and its properties, monotone, limited sequences. Geometric and arithmetic progressions.

Simple and compound interest. Accumulation and discounting.

Continuous accrual of interest.

Cobweb model of the market for one product. Price sequence and its convergence.

The concept of number series. Convergence of the series. The sum of the series. Eternal annuity. Limit of a function at a point and at infinity. One-sided limits.

Infinitesimal and Infinitely Large Functions. The first and second remarkable limits. Comparison of infinitely large and infinitely small functions. Equivalent functions and their use in calculating limits.

Continuity of a function at a point and on a set. Properties of continuous functions. Discontinuity points and their classification. Examples of continuous and discontinuous functions in the economy: cost function, dependence of the tax rate on income (the case of proportional and progressive tax).

Asymptotes of the graph of a function. Asymptotic behavior of Tornquist demand functions.

**Thematic component 3. Differential calculus of one-variable function.**

Derivative of a function, its geometric meaning, properties of the derivative. Derivative of a composite and implicitly defined function. Marginal and average values ​​in the economy: marginal and average costs, marginal and average labor productivity.

Average and point elasticity of function. Price elasticities of supply and demand, income elasticity of demand.

Differentiability of a function, the first differential and its geometric meaning. Approximate calculations using the differential. Fundamental theorems of differential calculus: Fermat's lemma, Rolle's and Lagrange's theorems. L'Hôpital's rule for resolving indeterminate forms.

Monotonicity of the function. Monotonicity condition. Extremum function. Necessary and sufficient conditions for an extremum. Profit maximization problem. Modeling tax revenues to the budget. Laffer curve.

The largest and smallest value of the function on the segment.

Higher-order derivatives and differentials. Taylor's formula. Maclaurin's formula. Expansion of elementary functions according to the Maclaurin formula.

Convexity of the function graph. Inflection points.

Full study of a function and plotting a graph by the methods of differential calculus.

**Thematic component 4. Integral calculus of one-variable function.**

Antiderivative of a function. Indefinite integral. Basic methods of integration: change of variable, integration by parts. Integration of rational and trigonometric functions.

Definite integral. The Newton-Leibniz formula and its application. Production output for a certain time at a given law of instantaneous production capacity.

Average value of the function. Average labor productivity, average return on capital.

Improper integrals. Poisson's integral.

**Thematic component 5. Multi-variable functions.**

Space *Rn*. Sets in the space *Rn*. Functions of several variables (multi-variable functions). Examples of functions of several variables in economics: utility function, multi-factor production functions (multiplicative, Cobb-Douglas). Ways to define a function of several variables. Function level surfaces (lines). Indifference curves and isoquants.

Limit and continuity of a function of several variables.

Partial derivatives of functions of several variables.

Differentiability and differential of a function of several variables.

Average and marginal labor productivity and capital productivity. Labor and capital elasticity coefficients of output. Marginal rates of substitution of factors of production.

Derivative of a composite function. Directional derivative and gradient.

Local extremum of a function of several variables. Necessary conditions for a local extremum. A sufficient condition for the case of two independent variables.

Conditional extremum. Substitution method. Lagrange multiplier method. The consumer choice problem, the economic meaning of the Lagrange multipliers.

Global extremum. Cost minimization and profit maximization of a multi-product firm.

Multiple integrals. Reduction of a multiple integral to a repeated one.

**Thematic component 6. Differential equations.**

Social and economic problems that lead to differential equations.

General solution of a differential equation. Particular solutions of a differential equation. Cauchy problem.

Separable equations. First order homogeneous equations. Linear equation of the first order. Bernoulli's equation.

Linear differential equations with constant coefficients.

Sustainability of the solution. Stability criterion.

**Part 2. Linear algebra**

**Thematic component 7. Vectors and matrices.**

Arithmetic vectors and their use in economics. Geometric interpretation of vectors. Linear operations on vectors. Inner product of vectors. Examples of the inner product in economics. Vector length. Angle between vectors.

Matrices and their types. Linear operations on matrices. Matrix transposition. Product of matrices. Properties of operations on matrices.

Elementary transformations over rows and columns of matrices. A theorem on reducing an arbitrary matrix to a row-echelon form and reduced row-echelon form. The rank of the matrix. Non-degeneracy of square matrices.

Inverse matrix. Properties of the inverse matrix. Calculation of the inverse matrix using elementary transformations.

Determinant of a square matrix. Minors and algebraic complements. Decomposition of the determinant by row or column. Determinant properties.

Criterion for non-degeneracy of a matrix. Calculation of the determinant using elementary transformations.

**Thematic component 8. Systems of linear algebraic equations.**

System of linear algebraic equations (SLAE). Homogeneous and non-homogeneous system of linear equations. Solution of a system of linear equations. Equivalence of systems of linear equations. Compatible and definite systems of linear equations. Kronecker-Capelli theorem.

Study and solution of a system of linear equations by the Jordan-Gauss method. General solution of a SLAE. Particular solutions of SLAE. Basic solutions of SLAE.

Fundamental system of solutions to a homogeneous system of equations. General solutions of homogeneous and non-homogeneous systems, the relationship between them.

Straight lines on a plane. Straight lines and planes in space.

Systems of linear algebraic inequalities and their use in economics: budget sets, restrictions on the use of resources.

Search for non-negative basic solutions of a system of linear equations.

Simplex transformations.

**Thematic component 9. Linear space.**

Linear (vector) space. Linear dependence (independence) of the vector system. Basis and dimension of linear space. Vector coordinates in a given basis. Vector coordinate transformation when changing the basis.

**Thematic component 10. Linear transformations and quadratic forms.**

Linear transformations of the space Rn (linear operators). Linear operator matrix. Transformation of the matrix of a linear operator when changing the basis.

Eigenvalues of the matrix. Characteristic polynomial of a matrix.

Eigenvectors of the matrix.

Linear exchange model (international trade model).

Symmetric matrices and quadratic forms. Reduction of a quadratic form to normal and canonical forms. Second order curves.

**Thematic component 11. Linear programming.**

Examples of linear optimization models in economics. Linear production task. Statement and various forms of writing a linear programming problem. Geometric interpretation of a linear programming problem.

The canonical form of the linear programming problem. Feasible solutions. Properties of the feasible set. Algorithm of the simplex method for linear programming.

Simplex method as a method of directed enumeration of basic feasible solutions. Optimality criterion. Economic interpretation of the linear programming problem, simplex method, simplex estimates.

Symmetrical pair of dual problems. Economic interpretation of the dual problem.

The main inequality of the theory of duality, its economic interpretation. Small duality theorem. A sufficient condition for the optimality of a pair of mutually dual problems. The first and second main theorems of duality, their geometric and economic interpretation.

An asymmetric pair of dual problems.

The third main theorem of duality, its geometric and economic interpretation. Stability region of dual estimates.

Transportation problem. The task is dual to the transportation one. A closed transportation problem and its solution by the method of potentials. Economic interpretation of cell estimates, supplier and consumer potentials.

Degenerate transportation problem. Fictitious supplies. Open transportation problem, fictitious suppliers and consumers. Mandatory and prohibited deliveries.

This subsection lists the academic subject itemized thematic contentin the form of a text. The text volume should not exceed 3 pages.

 **6. List of teaching and methodological materials needed for the students self-study**

**6.1. List of questions for student self-study and types of out-of-class activities**

The section lists types of out-of-class activities that correspond to items in the subject content description.

There is a list of questions the students should answer while working independently.

|  |  |  |
| --- | --- | --- |
| **Itemized subject content**  | **Questions the students should answer within the self-study process**  | **Types of out-of-class activities**  |
| **1. Number sets and functions** | Properties of one-variable functions | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson. |
| **2. Limits and continuity** | Cobweb model of the market for one product | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson.  |
| **3. Differential calculus of one-variable function.** | Proofs of main theorems. | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson.  |
| **4. Integral calculus of one-variable function.** | Average labor productivity, average return on capital. | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson. |
| **5. Multi-variable functions.** | Cost minimization and profit maximization of a multi-product firm. | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson. |
| **6. Differential equations.** | Sustainability of the solution. Stability criterion. | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson. |
| **7. Vectors and matrices.** | Properties of determinant. | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson. |
| **8. Systems of linear algebraic equations.** | Simplex transformations. | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson. |
| **9. Linear space.** | Solution of the tasks. | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson. |
| **10. Linear transformations and quadratic forms.** | Curves of the second order. | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson. |
| **11. Linear programming.** | The theorems of duality | Work with educational literature and tutorial videos. Analysis of questions on the topic of the lesson. |

 Table 3

**6.2. List of questions/assignments/topics for students’ preparation to formative assessment**

**Templates of questions for the exam preparation**

1. Local extrema of multi-variable functions.

2. Conditional extremum.

3. Global extremum.

4. Double integrals. Reduction to the repeated integral.

5. General solutions of differential equations.

6. Equations with separable values.

7. Linear differential equations with constant coefficients.

8. Arithmetic vectors. Matrices. Linear operations with matrices. Transposition.

9. Elementary transformations of matrix rows and columns.

10. Reduction of an arbitrary matrix to a row-echelon form.

11. Rank of a matrix. Non-degenerate quadratic matrix.

12. Determinant of a matrix.

13. Inverse matrix.

14. System of linear algebraic equations.

15. Linear (vector) space.

16. Linear independent set of vectors.

17. Basis and dimension of a linear space.

18. Linear transformations of *Rn*

19. Eigenvalues and eigenvectors.

20. Reduction of a quadratic form to normal and canonical forms.

The section may include the following:

Templates of questions the students need to answer when preparing for a test; templates of assignments that might be found in tests;

Templates of questions the students need to answer when preparing for performing an analysis or making a calculation;

Templates of topics for essays, summaries, creative home assignments;

Templates of questions the students need to answer when preparing for business games, roundtable discussion sessions, debates (based on a certain topic and relevant recommendations) conducted during a seminar; a list of assignments that should be done by a team of students when preparing for a co-authored work presentation.

Other types of assessment.

**7. Mandatory and optional reading list**

Legal regulations. The list of legal regulations adopted at the international and federal levels, including international treaties ratified by the Russian Federation, Constitution of the Russian Federation, codes, laws of the Russian Federation, federal laws, decrees of the President of Russia, resolutions of the Government of Russia, etc. The list should not exceed 10 entries.

**7.1. Mandatory**

Stuart J. *Calculus* / 8th edition, Cengage learning, 2015

Anton H., Rorres C. *Elementary linear algebra* / 11th edition, Wiley, 2014

**7.2. Optional**

Adams R., Essex C.*Calculus. A complete course /* 9th edition, Pearson, 2018

**8. List of IT resources, incl. the list of software, information and reference systems (as appropriate).**

**8. 1. Software:**

1. Windows OS;

2. Microsoft Office software.

**8.2. Databases and information and reference systems**

1. Information and education portal of the Financial University http://portal.ufrf.ru/.

2. Library of digital resources of the Financial University: http://elib.fa.ru/

**8.3. Certified software/hardware used for information protection**

ESET Endpoint Security antivirus software.

**Title page template**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(full name of education institution/branch)

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(name of department/teaching department)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(full name of author/authors)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(subject name)

**SYLLABUS**

***Level of Study:*** *Bachelor’s Degree / ~~Master’s Degree / Postgraduate Degree~~ \*to be chosen\**

***Field of Study:*** *\*to be defined\**

***Study Program:*** *\*to be defined\**

1. To be filled in when the updated Financial University educational standards and federal state educational standards of higher education “3++” are implemented. [↑](#footnote-ref-1)
2. Skills are described when the Financial University educational standards of the 1st generation and federal state educational standards of higher education “3+” are implemented. [↑](#footnote-ref-2)