**Federal state educational budgetary institution**

**of higher education**

**"FINANCIAL UNIVERSITY**

**UNDER THE GOVERNMENT OF THE RUSSIAN FEDERATION"**

**(Financial University)**

**Department of Mathematics**

Postovalova Galina Alexandrovna

**Mathematics**

**SYLLABUS**

***Level of Study:*** *Bachelor’s Degree*

***Field of Study:*** *38.03.02 - "Management"*

**Moscow 2021**

**Federal state educational budgetary institution**

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|  |  |
| --- | --- |
|  | **Approved**  Rector  \_\_\_\_\_\_\_\_\_\_\_\_M.A. Eskindarov  " "2021 |

Postovalova G.A.

**Mathematics**

**SYLLABUS**

***Level of Study:*** *Bachelor’s Degree*

***Field of Study: 38.03.02 - "Management"***

*Recommended by the Scientific Council*

*of the Faculty of Information Technology and Big Data Analysis*

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**Москва 2021**

**Reviewer:** Zadadaev S.A. Ph.D., Associate Professor, Department of Mathematics

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Discipline "Mathematics" is a discipline of the Module of Mathematics and Computer Science of the training direction 38.03.02 "Management" ( for all study programs in English).

The syllabus of the discipline indicates its purpose, its position in the structure of the educational program, requirements for the results of mastering the discipline, the content of the program, the topics of practical classes, forms of self- study, assessment tools for current control and intermediate certification, educational, methodological and information support.

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***Postovalova Galina Alexandrovna***

**Mathematics**

***Syllabus***

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1. **Name of a Subject –** «Mathematics».

**2. Mapping of learning outcomes (list of competences), with the relevant indicators described and subject learning outcomes indicated**

The subject "Mathematics" provides necessary tools to form the following competencies: PKN-2, UK-4.

|  |  |  |  |
| --- | --- | --- | --- |
| **Competence code** | **Competence** | **Competence development indicators[[1]](#footnote-2)** | **Learning outcomes (skills, abilities, and knowledge) correlated with competencies/indicators of competence achievement** |
| **PKN-2** | The ability to apply mathematical methods to solve standard professional problems, to interpret the obtained mathematical results. | 1. Demonstrates knowledge of mathematical methods used in management. | ***Know:*** fundamental concepts, ideas of algebra and geometry, mathematical analysis.  ***Be able to*:** apply mathematical methods for setting and solving analysis problems when evaluating the choice of optimal ways and methods to achieve goals in management. |
| 2. Applies mathematical methods and models to justify management decision - making. | ***Know:*** fundamental concepts, ideas of algebra and geometry, mathematical analysis.  ***Be able to:*** apply appropriate mathematical algorithms and methods for modeling management tasks. |
| 3. Consciously interprets the results obtained while using mathematical models. | ***Know:*** basic fundamental mathematical ideas, concepts, and principles of mathematical modeling.  ***Be able to:*** apply the tools of modern mathematics to analyze the results of studying mathematical models of financial and economic problems and be able to draw on this basis quantitative and qualitative conclusions and recommendations for making financial and economic decisions. |
| **UK-4** | Ability to use application software in solving professional tasks. | 1. Uses the main methods and means of obtaining, presenting, saving and processing data. | ***Know:*** the main methods of collecting, processing information, methods of mathematical analysis of data.  ***Be able to:*** apply mathematical methods for setting and solving analysis problems when evaluating the choice of optimal ways and methods to achieve goals. |
| 2. Demonstrates skills in professional application software packages. | ***Know:*** fundamental concepts, ideas and tools of algebra and geometry, mathematical analysis.  ***Be able to:*** select adequate mathematical methods and models for setting, solving and analyzing the results obtained in specific applied problems using application software packages. |
| 3. Selects the required application software, depending on the task to be solved. | ***Know:*** fundamental concepts, ideas and tools of algebra and geometry, mathematical analysis.  ***Be able to:*** apply mathematical methods for setting, solving and interpreting the results obtained in the tasks of modeling and describing professional activities using application software packages. |
| 4. Uses application software to solve certain application tasks. | ***Know:*** fundamental concepts, ideas of algebra and geometry, mathematical analysis.  ***Be able to:*** apply mathematical methods and applied software for setting and making financial and economic decisions. |

# 3. **Place of the subject in the curriculum**

# The discipline "*Mathematics*" is included in the Module of Mathematics and Computer Science (the training direction 38.03.02 "Management").

# The study of the discipline "*Mathematics*" is based on the knowledge gained from a school mathematics course or related subjects of secondary vocational education. The discipline "Mathematics" is the theoretical basis for all disciplines of the module of Mathematics and Computer Science. Mathematical concepts and methods are used later in the study of professional disciplines and disciplines of this profile.

# 4. **Workload in credits and academic hours, with class work (lectures and seminars) and self-study indicated**

# The total workload of the course is 6 credit units.

# Types of intermediate certification - test, exam.

# Type of current control - 2 classroom tests (1, 2 semesters).

Full-time education

|  |  |  |  |
| --- | --- | --- | --- |
| **Type of academic activities on the discipline** | **Total**  **(in credits and hours)** | **Semester 1 (in hours)** | **Semester 2 (in hours)** |
| **Total training load** | **6 cr.=216 hr.** | **108/108** | **108/108** |
| ***Class work*** | ***100/32*** | ***50/16*** | ***50/16*** |
| *Lectures* | ***32/6*** | ***16/2*** | ***16/4*** |
| *Seminars, incl.* | ***68/26*** | ***34/14*** | ***34/12*** |
| *Interactive classes* | ***60/26*** | ***30/14*** | ***30/12*** |
| ***Self-study*** | ***116/184*** | ***58/92*** | ***58/92*** |
| Current control. | | **Test work** | **Test work** |
| Intermediate control. | | **Test/**  **Test** | ***Exam*/**  ***Exam*** |

**Part-time training (profile: Management and Business management)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Type of academic activities on the discipline** | **Total**  **(in credits and hours)** | **Semester 1 (in hours)** | **Semester 2 (in hours)** |
| **Total training load** | **6 cr.=216 hr.** | **108/-** | **108/-** |
| ***Class work*** | ***68*/-** | ***34*/-** | ***34*/-** |
| *Lectures* | ***32*/-** | ***16*/-** | ***16*/-** |
| *Seminars, incl.* | ***36*/-** | ***18*/-** | ***18*/-** |
| *Interactive classes* | ***36*/-** | ***18*/-** | ***18*/-** |
| ***Self-study*** | ***148*/-** | ***74*/-** | ***74*/-** |
| Current control. | | **Test work/-** | **Test work/-** |
| Intermediate control. | | **Test/-** | ***Exam*/-** |

**Accelerated learning (profiles: Audit and Internal control; Taxes and Taxation)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Type of academic activities on the discipline** | **Total**  **(in credits and hours)** | **Semester 1 (in hours)** | **Semester 2 (in hours)** |
| **Total training load** | **6 cr.=216 hr.** | **72/-** | **72/-** |
| ***Transfer*** | ***2 з.е. = 72 ч.*** | ***1 з.е. = 36 ч.*/-** | ***1 з.е. = 36 ч.*/-** |
| ***Class work*** | ***68*/-** | ***34*/-** | ***34*/-** |
| *Lectures* | ***32*/-** | ***16*/-** | ***16*/-** |
| *Seminars, incl.* | ***36*/-** | ***18*/-** | ***18*/-** |
| *Interactive classes* | ***36*/-** | ***18*/-** | ***18*/-** |
| ***Self-study*** | ***76*/-** | ***38*/-** | ***38*/-** |
| Current control. | | **Test work/-** | **Test work/-** |
| Intermediate control. | | **Test/-** | ***Exam*/*-*** |

**Accelerated learning (profiles: State and Municipal Finance; Financial Markets and Banks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Type of academic activities on the discipline** | **Total**  **(in credits and hours)** | **Semester 1 (in hours)** | **Semester 2 (in hours)** |
|  | **6 з.е.=216 ч.** | **72/144** | **72/-** |
| **Total training load** | ***2 з.е. = 72 ч.*** | ***1 з.е. = 36 ч./***  ***2 з.е. = 72 ч.*** | ***1 з.е. = 36 ч./-*** |
| ***Transfer*** | ***68/20*** | ***34/20*** | ***34/-*** |
| *Lectures* | ***32/6*** | ***16/6*** | ***16/-*** |
| *Seminars, incl.* | ***36/14*** | ***18/14*** | ***18/-*** |
| *Interactive classes* | ***36/14*** | ***18/14*** | ***18/-*** |
| ***Self-study*** | ***76/124*** | ***38/124*** | ***38/-*** |
| Current control. | | **Test work** | **Test work/-** |
| Intermediate control. | | **Test/Exam** | **Exam/-** |

**Accelerated learning (profiles: Accounting, Analysis and Audit)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Type of academic activities on the discipline** | **Total**  **(in credits and hours)** | **Semester 1 (in hours)** | **Semester 2 (in hours)** |
| **Total training load** | **6 з.е.=216 ч.** | **-/144** | **-/-** |
| ***Transfer*** | ***2 з.е. = 72 ч.*** | ***-/2 з.е. = 72 ч.*** | ***-/-*** |
| ***Class work*** | ***-/20*** | ***-/20*** | ***-/-*** |
| *Lectures* | ***-/6*** | ***-/6*** | ***-/-*** |
| *Seminars, incl.* | ***-/14*** | ***-/14*** | ***-/-*** |
| *Interactive classes* | ***-/14*** | ***-/14*** | ***-/-*** |
| ***Self-study*** | ***-/124*** | ***-/124*** | ***-/-*** |
| Current control. | | **-/Test work** | **-/-** |
| Intermediate control. | | **-/Exam** | **-/-** |

# **5. Subject content (with the thematic components indicated).**

# 5.1. **Subject content**

**Part 1. Calculus**

**Topic 1. Numerical sets and functions**

Elements of set theory. Quantifiers. Operations on sets: union, intersection, difference, complement. Finite, countable, and uncountable sets. Bounded and unbounded sets**.**

Sets of natural, integer, rational and real numbers. Complex numbers and operations on them. The modulus and the argument of a complex number. Algebraic and trigonometric forms of notation of complex numbers.

The concept of a function. A numeric function of a single variable. Methods for setting functions. Graph of a function. Properties of functions of a single variable: even, odd, monotonic, periodic and unlimited functions. Concavity and convexity.

Functions in economics: utility functions, one-factor production functions, supply and demand functions. Average cost functions and the relationship between them (ATC = AVC + AFC).

**Topic 2. Limits and continuity**

Numerical sequences, the limit of a sequence and its properties, monotonic, bounded sequences. Geometric and arithmetic progressions.

Simple and compound interest. Accumulation and Discounting. Continuous accrual of interest.

Cobweb model of the market for a single product. Price sequence and its convergence.

The concept of numerical series. Convergence of the series. The sum of the series. Eternal annuity.

Limit of a function at a point and at infinity. One-sided limits.

Infinitesimal Function and Infinite Function. The first and second remarkable limits. Comparison of infinitely large and infinitely small functions. Equivalent functions and their application for calculating limits.

Continuity of a function at a point and on a set. Properties of continuous functions. Discontinuity points and their classification. Examples of continuous and discontinuous functions in the economy: cost function, dependence of the tax rate on income (the case of proportional and progressive tax).

Asymptotes of the graph of a function. Asymptotic behavior of Tornquist demand functions.

**Topic 3. Differential calculus of a single-variable function**

Derivative of a function, its geometric meaning, properties of the derivative. Derivative of a composite and implicitly defined function. Marginal and average values in the economy: marginal and average costs, marginal and average labor productivity.

Average and point elasticity of function. Price elasticities of supply and demand, income elasticity of demand.

Differentiability of a function, the first differential and its geometric meaning. Approximating using the differential. Fundamental theorems of differential calculus: Fermat's lemma, Rolle's and Lagrange's theorems. L'Hôpital's rule for resolving indeterminate forms.

Monotonicity of a function. The condition of monotonicity. Extremum of a function. Necessary and sufficient conditions for an extremum. Profit maximization problem. Modeling tax revenues to the budget. Laffer curve.

The largest and smallest values of a function on the segment.

Higher-order derivatives and differentials. Taylor's formula. Maclaurin's formula. Expansion of elementary functions according to the Maclaurin formula.

Convexity of the graph of a function. Inflection points.

Application of differential calculus for the analysis and plotting of a function.

**Topic 4. Integration calculus of a single-variable function.**

Antiderivative of a function. Indefinite integral. Basic methods of integration: change of variable, integration by parts. Integration of rational and trigonometric functions.

Definite integral. The Newton-Leibniz formula and its application. Production output for a certain time at a given law of instantaneous production capacity.

Average value of the function. Average labor productivity, average return on capital.

Improper integrals. Poisson's integral.

**Topic 5. Multi-variable functions**

Space *Rn*. Sets in the space *Rn*. Functions of several variables (multi-variable functions). Functions of several variables in economics: utility function, multi-factor production functions (multiplicative, Cobb-Douglas). Ways to define a function of several variables. Level surfaces (lines) of a function. Indifference curves and isoquants.

Limit and continuity of a function of several variables.

Partial derivatives of functions of several variables.

Differentiability and differentials of a function of several variables.

Average and marginal labor productivity and capital productivity. Elasticity coefficients of output in terms of labor and capital. Marginal rates of substitution of production factors .

Derivative of a composite function. Directional derivative and gradient.

Local extremum of a function of several variables. Necessary conditions for a local extremum. A sufficient condition for the function of two independent variables.

Conditional extremum. Method of substitution. Lagrange multiplier method. The consumer choice problem, the economic sense of the Lagrange multipliers.

Global extremum. Cost minimization and profit maximization of a multi-product firm.

Multiple integrals. Iterated Integrals.

**Topic 6. Differential equations**

Social and economic problems that require differential equations to be solved.

General solution of a differential equation. Particular solutions of a differential equation. Cauchy problem.

Separable equations. First order homogeneous equations. Linear equation of the first order. Bernoulli's equation.

Linear differential equations with constant coefficients.

Sustainability of the solution. Stability criterion.

**Part 2. Linear algebra**

**Topic 7*.* Vectors and matrices**

Arithmetic vectors, application in economics. Geometric interpretation of vectors. Linear operations on vectors. Inner product of vectors. Examples of the inner product in economics. Vector length. Angle between vectors.

Matrices and their types. Linear operations on matrices. Matrix transposition. Product of matrices. Properties of operations on matrices.

Elementary transformations over rows and columns of matrices. Reducing an arbitrary matrix to a row-echelon form and reduced row-echelon form (theorem). The rank of the matrix. Non-degeneracy of square matrices.

Inverse matrix. Properties of the inverse matrix. Calculation of the inverse matrix using elementary transformations.

Determinant of a square matrix. Minors and algebraic complements. Decomposition of the determinant by row or column. Determinant properties.

Criterion for non-degeneracy of a matrix. Calculation of the determinant using elementary transformations.

**Topic 8*.* Systems of linear algebraic equations**

System of linear algebraic equations (SLAE). Homogeneous and non-homogeneous system of linear equations. Solution of a system of linear equations. Equivalence of systems of linear equations. Compatible and definite systems of linear equations. Kronecker-Capelli’s theorem.

Jordan-Gauss method for solving a system of linear equations. General solution of SLAE. Particular solutions of SLAE. Basic solutions of SLAE.

Fundamental system of solutions to a homogeneous system of equations. General solutions of homogeneous and non-homogeneous systems, the relationship between them.

Straight lines on a plane. Straight lines and planes in space.

Systems of linear algebraic inequalities, applying in economics: budget sets, restrictions on the use of resources.

Defining non-negative basic solutions of equations system.

Simplex transformations.

**Topic 9. Linear space**

Linear (vector) space. Linear dependence (independence) of the vector system. Basis and dimension of linear space. Vector coordinates in a given basis. Vector coordinate transformation when changing the basis.

**Topic 10. Linear transformations and quadratic forms**

Linear transformations of the n**–**space (linear operators). Matrix of linear operator. Transformation of the matrix of a linear operator when changing the basis.

Eigenvalues of a matrix. Characteristic polynomial of a matrix.

Eigenvectors of a matrix.

Linear exchange model (international trade model).

Symmetric matrices and quadratic forms. Reduction of a quadratic form to normal and canonical forms. Second order curves.

**Topic 11. Linear programming**

Samples of linear optimization models in economics. Linear production task. Statement and various forms of writing a linear programming problem. Geometric interpretation of a linear programming problem.

The canonical form of the linear programming problem. Feasible solutions. Properties of the feasible set. Algorithm of the simplex method for linear programming.

Simplex method as a method of directed enumeration of basic feasible solutions. Optimality criterion. Economic interpretation of the linear programming problem, simplex method, simplex estimates. Statement and various forms of representation a linear programming problem. Geometric interpretation of a linear programming problem.

The canonical representation of the linear programming problem. Feasible solutions. Properties of the feasible set. Algorithm of the simplex method for linear programming.

Simplex method as a method of directed enumeration of basic feasible solutions. Optimality criterion. Economic interpretation of the linear programming problem, simplex method, simplex estimates.

Symmetrical pair of dual problems. Economic interpretation of the dual problem.

The main inequality of the theory of duality, its economic interpretation. Small duality theorem. A sufficient condition for the optimality of a pair of mutually dual problems. The first and second main theorems of duality, their geometric and economic interpretation.

An asymmetric pair of dual problems.

# The third basic duality theorem, its geometric and economic interpretation. The area of stability of dual estimates.

Transport problem. The problem that is dual to the transport one. A closed transport problem and its solution by the method of potentials. Economic interpretation of cell estimates, supplier and consumer potentials.

Degenerate transport problem. Fictitious supplies. Open transport problem, fictitious suppliers and consumers. Mandatory and prohibited deliveries.

# 5.2. Curriculum - thematic plan

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| № | **The name of topic (section)** | **Hours** | | | | | | | **Forms of current control** |
| **Total (hours)** | **Classroom work** | | | | | **Self-study work** |
| total | | Lectures | Seminars | Classes in interactive forms |
| **Section 1. Calculus.** | | | | | | | | | |
|  | Numerical sets and functions. | 9/10 | 3/2 | 1/0,5 | | 2/1,5 | 2/1,5 | 6/8 | Self-study. Participation in solving problems in  practical classes. The discussion  of the solved problems. Test work. |
|  | Limit and continuity | 17/25,5 | 9/3,5 | 3/0,5 | | 6/3 | 5/3 | 8/22 |
|  | Differential calculus of functions of one variable | 29/30 | 19/4 | 5/0,5 | | 14/3,5 | 13/3,5 | 10/26 |
|  | Integral calculus of the functions of one variable | 25/30 | 15/4 | 5/0,5 | | 10/3,5 | 8/3,5 | 10/26 |
|  | Functions of Several variables | 34/31,5 | 16/3,5 | 6/0,5 | | 10/3 | 8/3 | 18/28 |
|  | Series and Differential equations | 16/12 | 6/2 | 2/0,5 | | 4/1,5 | 4/1,5 | 10/10 |
| **Section 2 . Linear Algebra** | | | | | | | | | |
|  | Vectors and matrices | 16/14 | 6/2 | 2/0,5 | | 4/1,5 | 3/1,5 | 10/12 | Self-study. Participation in solving problems in  practical classes. The discussion  of the solved problems. problems. Test work. |
|  | Systems of linear equations and inequalities | 16/14,5 | 6/2,5 | 2/0,5 | | 4/2 | 4/2 | 10/12 |
|  | Linear space | 3,5/6 | 1,5/  1,75 | 0,5/  0,25 | | 1/1,5 | 1/1,5 | 2/4,25 |
|  | Linear transformations and quadratic forms | 17,5/  14,5 | 7,5/  2,75 | 2,5/  0,75 | | 5/2 | 5/2 | 10/  11,75 |
|  | Linear programming | 33/28 | 11/4 | 3/1 | | 8/3 | 7/3 | 22/24 |
|  | **Total** | **216/**  **216** | **100/32** | **32/6** | | **68/26** | **60/26**  **60%/**  **81%** | **116/184** |  |

**Part-time training (profile: Management and Business management)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| № | **The name of topic (section)** | **Hours** | | | | | | **Forms of current control** |
| **Total (hours)** | **Classroom work** | | | | **Self-study work** |
| total | Lectures | Seminars | Classes in interactive forms |
| **Section 1. Calculus** | | | | | | | | |
| 1 | Numerical sets and functions. | 6 | 2 | 1 | 1 | 1 | 4 | Self-study. Participation in solving problems in  practical classes. The discussion  of the solved problems. Test work. |
| 2 | Limit and continuity | 18 | 6 | 3 | 3 | 3 | 12 |
| 3 | Differential calculus of functions of one variable | 27 | 11 | 5 | 6 | 6 | 16 |
| 4 | Integral calculus of the functions of one variable | 27 | 11 | 5 | 6 | 5 | 16 |
| 5 | Functions of Several variables | 31 | 11 | 5 | 6 | 5 | 20 |
| 6 | Series and Differential equations | 16 | 4 | 2 | 2 | 2 | 12 |
| **Section 2. Linear Algebra** | | | | | | | | |
| 7 | Vectors and matrices | 18 | 4 | 2 | 2 | 2 | 14 | Self-study. Participation in solving problems in  practical classes. The discussion  of the solved problems. Test work. решенных задач. Контроль  ная работа. |
| 8 | Systems of linear equations and inequalities | 18 | 4 | 2 | 2 | 2 | 14 |
| 9 | Linear space | 3 | 1 | 0,5 | 0,5 | 0,5 | 2 |
| 10 | Linear transformations and quadratic forms | 19 | 5 | 2,5 | 2,5 | 2,5 | 14 |
| 11 | Linear programming | 33 | 9 | 4 | 5 | 5 | 24 |
|  | **Total** | **216** | **68** | **32** | **36** | **36**  **53%** | **148** |  |

**Accelerated learning (profiles: State and Municipal Finance; Financial Markets and Banks)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| № | **The name of topic (section)** | **Hours** | | | | | | **Forms of current control** |
| **Total (hours)** | **Classroom work** | | | | **Self-study work** |
| total | Lectures | Seminars | Classes in interactive forms |
| **Section 1. Calculus** | | | | | | | | |
| 1 | Numerical sets and functions. | 4/6 | 2/1 | 1/0,5 | 1/0,5 | 1/0,5 | 2/5 | Self-study. Participation in solving problems in  practical classes. The discussion  of the solved problems. Test work. |
| 2 | Limit and continuity | 12/  17,5 | 6/2,5 | 3/0,5 | 3/2 | 3/2 | 6/15 |
| 3 | Differential calculus of functions of one variable | 19/  19,5 | 11/  2,5 | 5/0,5 | 6/2 | 6/2 | 8/17 |
| 4 | Integral calculus of the functions of one variable | 19/  18,5 | 11/  1,5 | 5/0,5 | 6/1 | 5/1 | 8/17 |
| 5 | Functions of Several variables | 21/20 | 11/3 | 5/1 | 6/2 | 5/2 | 10/17 |
| 6 | Differential equations | 10/8,5 | 4/1,5 | 2/0,5 | 2/1 | 2/1 | 6/7 |
| **Section 2. Linear Algebra** | | | | | | | | |
| 7 | Vectors and matrices | 11/  10,5 | 4/1,5 | 2/0,5 | 2/1 | 2/1 | 7/9 | Self-study. Participation in solving problems in  practical classes. The discussion  of the solved problems. Test work. |
| 8 | Systems of linear equations and inequalities | 11/11 | 4/2 | 2/0,5 | 2/1,5 | 2/1,5 | 7/9 |
| 9 | Linear space | 2/3 | 1/  0,75 | 0,5/  0,25 | 0,5/0,5 | 0,5/0,5 | 1/2,25 |
| 10 | Linear transformations and quadratic forms | 12/9 | 5/  1,25 | 2,5/  0,75 | 2,5/0,5 | 2,5/0,5 | 7/  7,75 |
| 11 | Linear programming | 23/  20,5 | 9/2,5 | 4/0,5 | 5/2 | 5/2 | 14/18 |
|  | **Total** | **144/**  **144** | **68/**  **20** | **32/6** | **36/14** | **36/14**  **53%/**  **70%** | **76/124** |  |

**Accelerated learning (profiles: Accounting, Analysis and Audit)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| № | **The name of topic (section)** | **Hours** | | | | | | **Forms of current control** |
| **Total (hours)** | **Classroom work** | | | | **Self-study work** |
| total | Lectures | Seminar[[2]](#endnote-2)s | Classes in interactive forms |
| **Section 1. Calculus** | | | | | | | | |
| 1 | Numerical sets and functions. | -/6 | -/1 | -/0,5 | -/0,5 | -/0,5 | -/5 | Self-study. Participation in solving problems in  practical classes. The discussion  of the solved problems. Test work. |
| 2 | Limit and continuity | -/17,5 | -/2,5 | -/0,5 | -/2 | -/2 | -/15 |
| 3 | Differential calculus of functions of one variable | -/19,5 | -/2,5 | -/0,5 | -/2 | -/2 | -/17 |
| 4 | Integral calculus of the functions of one variable | -/18,5 | -/1,5 | -/0,5 | -/1 | -/1 | -/17 |
| 5 | Functions of Several variables | -/20 | -/3 | -/1 | -/2 | -/2 | -/17 |
| 6 | Differential equations | -/8,5 | -/1,5 | -/0,5 | -/1 | -/1 | -/7 |
| **Section 2. Linear Algebra** | | | | | | | | |
| 7 | Vectors and matrices | -/10,5 | -/1,5 | -/0,5 | -/1 | -/1 | -/9 | Self-study. Participation in solving problems in  practical classes. The discussion  of the solved problems. Test work. |
| 8 | Systems of linear equations and inequalities | -/11 | -/2 | -/0,5 | -/1,5 | -/1,5 | -/9 |
| 9 | Linear space | -/3 | -/  0,75 | -/  0,25 | -/0,5 | -/0,5 | -/2,25 |
| 10 | Linear transformations and quadratic forms | -/9 | -/  1,25 | -/  0,75 | -/0,5 | -/0,5 | -/  7,75 |
| 11 | Linear programming | -/  20,5 | -/2,5 | -/0,5 | -/2 | -/2 | -/18 |
|  | **Total** | **-/**  **144** | **-/**  **20** | **-/6** | **-/14** | **-/14**  **-/70%** | **-/124** |  |

# **5.3. Content of seminars, workshops**

|  |  |  |
| --- | --- | --- |
| **The name of topic (section)** | **List of questions to discuss in seminary, practice sessions, recommended sources from section 8.9 (indicated section and serial number of the source)** | **Forms of classes** |
| **1. Numerical sets and functions.** | Operations on numerical sets .The study of numerical sets for boundedness. Finding the complex roots of a polynomial. Complex numbers and operations on them. The module and the argument of a complex number. Algebraic and trigonometric forms of notation of complex numbers.  Functions in economics: utility functions, one-factor production functions, supply and demand functions. Average cost functions and the relationship between them (ATC = AVC + AFC). | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **2. Numerical sequences. The limit of a sequence.** | Cobweb model of the market for a single product. Calculating limits of a sequence. Investigation of series for convergence. Solving problems: compound interest, continuous accrual of interest, eternal annuity. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **3. Limit of a function** | Limit of a function at a point and at infinity. Calculating One-sided limits. Solving problems on comparison of infinitely large and infinitely small functions. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **4. Continuity of a function. The points of discontinuity. Asymptotes.** | Finding discontinuity points and types of discontinuity. Finding the asymptotes of the graph of the function. Plotting the Tornquist demand functions and finding their asymptotes. | Solving problems in an interactive form, checking self-study and analyzing mistakes.  *Test work № 1.* |
| **5. The Derivative. Marginal and average values in the economy. Medium and point elasticity.** | Calculating the derivatives of a function of a single variable. Finding the tangent to the graph of the function. Calculating the marginal values in economics and their interpretation. Calculating the average and point elasticity of the functions of supply and demand for price, the elasticity of demand for income. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **6. Differentiability of the function. Differential. Derivatives of complex and implicitly defined functions.** | Approximate calculation of the function value using the differential. Calculation of derivatives of composite and implicitly defined functions. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **7. Basic theorems of differential calculus. The Lopital's rule.** | Calculating limits using the Lopital's rule. Solving problems for finding intervals of monotonicity of a function. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **8. The study of functions using the first derivative: monotonicity, local extremes, the largest and smallest values on the segment.** | Finding the extremum points and extremums of a function of one variable. Solving problems for finding the largest and smallest values of a function on a segment. The task of maximizing profit. The task of maximizing tax revenue. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **9. Higher-order derivatives and differentials. Convexity and concavity of the function, inflection points. The Taylor and Maclaurin formulas.** | Calculating the derivatives and differentials of a second-order function. Finding the convexity/concavity intervals of the function and the inflection points. Solving problems based on the Taylor and Maclaurin formulas. Using them for approximate calculations. | РSolving problems in an interactive form, checking self-study and analyzing mistakes.  *Test work № 2.* |
| **10. Functions research and plotting.** | A complete study of the function and plotting its graph. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **11. Antiderivative and indefinite integral.** | Calculation of indefinite integrals by direct integration, by the change of variables, by the method of integration by parts. |  |
| **12. Methods for calculating indefinite integrals.** | Calculation of special classes of indefinite integrals. Solving problems for the integration of rational fractions. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **13. Definite integral and its application.** | Calculation of definite integrals by the Newton-Leibniz formula.  Calculating the areas of flat shapes. Finding the output of products for a certain time at a given law of instantaneous production capacity. Calculation of the average value of the function. Calculation of average labor productivity and average capital return. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **14. Improper integrals.** | Solving the problem of convergence (divergence) of improper integrals. Calculation of convergent improper integrals. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **15. Test №1** | Test No. 1 on topics 1-4 of the content of the discipline. | Control of problem solving. |
| **16. Space *Rn*. Sets in the space *Rn*. Functions of several variables.**  **Differentiation of Functions of several variables.** | Representation of sets in the spaces R2 and R3. The distance between points in space. Construction of surfaces and level lines, indifference curves and isoquants. Calculation of partial derivatives of a function of several variables and the derivative of a complex function. Calculation of average and marginal labor productivity and capital return. Calculation of the elasticity of output for labor and capital, the marginal rate of substitution of production factors. Calculation of the derivative of a composite function, the derivative in the direction and the gradient. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **17. Test.** | Intermediate certification in the form of a report on the topics of the discipline in the 1st semester. | Monitoring the implementation of test tickets. |
| **18. Local extremum of functions of several variables.** | Solving problems for finding local extremums of functions of several variables. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **19. Conditional extremum of functions of several variables.** | Solving problems for finding extremums subject to the constrate: the substitution method and the Lagrange multiplier method. The problem of consumer choice, the economic meaning of Lagrange multipliers. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **20. Global extremum of functions of several variables.** | Solving problems for finding the global extremum of a differentiable function on a closed bounded set. Minimizing costs and maximizing the profit of a multi-product company. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **21. Double Integrals.** | Calculation of multiple integrals. Reducing a multiple integral to a repeated one. | Solving problems in an interactive form, checking self-study and analyzing mistakes.  *Test work № 3.* |
| **22. The concept of a numerical series** | The concept of numerical series. Convergence of the series. The sum of the series. Eternal rent.  . | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **23. Second-order linear differential equations with constant coefficients** | Solving second-order linear differential equations with constant coefficients. Research of solutions for sustainability. | Solving problems in an interactive form, checking self-study and analyzing mistakes.  *Test work № 4.* |
| **24. Systems of linear algebraic equations.** | Solving systems of linear algebraic equations by the Jordan-Gauss method. Finding non-negative solutions of systems of linear algebraic equations. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **25. Vectors and operations on them.** | Arithmetic vectors and their application in economics. Geometric interpretation of vectors. Linear operations on vectors. The inner product of vectors. Examples of the inner product in economics. The length of the vector. Angle between the vectors. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **26. Matrices and operations on them.** | Solving problems on operations with matrices. Elementary transformations on rows and columns of matrices. Calculating the rank of the matrix. Calculation of the inverse matrix. Calculation of the determinant of the matrix. | Solving problems in an interactive form, checking self-study and analyzing mistakes.  *Test work № 5.* |
| **27. Linear space and linear transformations.** | Studying the system of vectors for linear dependence. Calculation of the coordinates of the vector when changing the basis. Solving problems for linear transformations. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **28. Eigenvalues and eigenvectors. A linear exchange model.** | Finding the eigenvalues and eigenvectors of the matrix. The study of a linear exchange model. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **29. Quadratic forms and second-order curves.** | Reduction of a quadratic form to a normal and canonical form. Determining the type of the second-order curve. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **30. Lines and planes. Convex sets** | Solving problems on straight lines on a plane and straight lines and planes in space. Finding the regions specified by the system of inequalities. | Solving problems in an interactive form, checking self-study and analyzing mistakes.  *Test work № 6.* |
| **31. Graphical method for solving linear programming problems** | Solving linear programming problems graphically. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **32. Linear models in economics. Transport task\*** | Solving linear programming problems by the simplex method. Economic interpretation of the simplex method and simplex estimates. Solving transport problems. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |
| **33. Test №2** | Test No. 2 on topics 5-11 of the content of the discipline. | Control of problem solving. |
| **34. Repetition.** | Preparing for the examination paper. | Solving problems in an interactive form, checking self-study and analyzing mistakes. |

# 6. **List of teaching and methodological materials needed for the students self-study**

# 6.1. List of questions assigned to self-mastering subject, forms of extra-auditing self-employment

|  |  |  |
| --- | --- | --- |
| **Name of the topics (sections) of the discipline.** | **The list of issues assigned to self-exploration** | **The list of issues assigned to self-exploration** |
| **Section 1. Calculus** | | |
| Numerical sets and functions | Properties of functions of a single variable. Solving problems. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |
| Limits and Continuity | Research of the web-like model of the market of one product. Solving tasks. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |
| Differential Calculus of a function of a single variable. | Proofs of the main theorems of differential calculus. Solving problems. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |
| Integration Calculus of a function of a single variable. | Solving problems. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |
| Functions of several variables. | Limit and continuity of functions of several variables. Solving problems. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |
| Numerical series and differential equations | Eternal rents. Models of natural growth. Models of economic dynamics. Solving problems. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |
| **Section 2. Linear algebra** | | |
| Systems of linear equations and inequalities. | Solving problems. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |
| Vectors and matrices | Properties of the determinant of the matrix. Solving problems. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |
| Linear space | Solving problems. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |
| Linear transformations and quadratic forms. | Solving problems. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |
| Linear programming. | The theory of duality.  Solving problems. | Activities with educational literature, preparation for seminars and practical classes, solving problems on the topics of seminars.  Performing tasks of the control work. |

# 6.2. List of questions, assignments, topics to prepare for current control

|  |  |  |  |
| --- | --- | --- | --- |
| **Name of the topics (sections) of the discipline.** | **The list of issues assigned to the preparation** | **Training Forms** |  |
| **Section 1. Calculus** | | |  |
| Numerical sets and functions | Arithmetic operations with complex numbers. Representation of a complex number in algebraic and trigonometric form. | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| Limits and Continuity | Calculation of the limits of the numerical sequence and functions at infinity and at a point. Finding the points of discontinuity and asymptotes of the graph of the function. | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| Differential calculus of functions of single variable. | Calculation of derivatives. Finding the limits with the Lopital rule.  The study of the function and plotting the graph (monotonicity intervals and extremes, convexity intervals and inflection points, asymptotes).Determining the largest and smallest values of a function on a segment | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| Integration calculus of functions of single variable. | Calculation of indefinite integrals by direct integration, by the change of variables, by the method of integration by parts. Calculation of definite integrals according to the Newton-Leibniz formula, convergent improper integrals, areas of flat figures. | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| Functions of several variables. | Calculation of partial derivatives, the derivative of a composite function, the directional derivative and the gradient.  Finding local and conditional extremes.  Determining the maximum and minimum values. Calculation of multiple integrals. | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| Numerical series. | The concept of numerical series. Convergence of the series. The sum of the series. Eternal rent. | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| Differential equations. | Solution of first-order differential equations and second-order linear differential equations with constant coefficients | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| **Section 2. Linear algebra** | | |  |
| Vectors and matrices | Solving problems for operations with vectors and matrices. Calculation of the rank of the matrix, the inverse of the matrix. the determinant of the matrix. | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| Systems of linear equations and inequalities | Solving systems of linear algebraic equations by the Jordan-Gauss method. Straight lines on a plane, straight lines and planes in space | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| Linear space | Investigation of a vectors system for linear dependence. The basis and dimension of the space. | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| Linear transformations and quadratic forms | Finding the eigenvalues and eigenvectors of the matrix. Solving problems on the sign-definiteness of a quadratic form. Transformation of a quadratic form to a canonical and normal form. | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |
| Linear programming | Solving linear programming problems graphically and with simplex method. | Study the material of the lecture, analysis of questions and tasks on the topic of the lesson;  study of literary sources recommended for the lesson. |  |

**Example of control work N1 (1 semester).**

1. Calculate the limit of the sequence .

2. Calculate the limit of the function .

3. The dependence of demand for a product on its price is expressed by the function . Find demand for goods, marginal demand and point elasticity of demand at a price of *p* = 3. What is the average elasticity of demand at a price, with an increase in price by 4%?

5.The total costs for the production of *q* units is expressed by the function . The demand function for these products is , where *р* – unit price.

1) Find a minimum of: a) total costs ; b) average costs .

2) Draw a graph of marginal costs .

3) Create a sales revenue function  from the sale of q units of goods at the price p.

4) Find the profit .

5) Draw income  and profit  graphs.

6. Explore the function  and plot the graph.

7. The productivity of one worker per day is described by a function where t is hours time, 0 ≤ t ≤ 8. Determine the output in 20 business days by a team of 6 people.

**Example of control work N2 (2 semester).**

1. Find the extreme values of the function .

2. The consumer utility function for two products is , where *x,* *y* − the number of goods purchased.

1) Determine the maximum utility of the goods if the consumer has a budget in *I* =2000, and the prices of goods are 15 and 5 respectively.

2) Plot the utility function graph.

3) Draw a valid set, indifference curves, and an optimal point.

4) Find the equation of the indifference curve on which the optimal consumer point is located.

5) Calculate the rate of replacement of the second product with the first at the optimal point.

5) Determine the demand function for the first product and plot its graph.

6) Calculate the elasticity of demand for the first product at a price at given

prices and a given consumer budget.

7) Explain the economic meaning of the indicators found.

3. Solve the differential equation .

4. The matrices are given  ,  и .

Find the matrix 

5. Find the determinant of the matrix , if the following matrix equality is satisfied

.

6. Solve a system of linear algebraic equations and find at least two of its basic non-negative solutions



7. Determine whether the international trade of the two countries is balanced, if the vector of national income vector of these countries is



andstructural matrix of trade is.

8. There are 100 kg of raw materials for the manufacture of two types of products. 2 kg is used on manufacturing one product of the first type, 4 kg of raw materials is used on manufacturing one product of the second type. Make a production plan that ensures the There are 100 kg of raw materials for the manufacture of two types of products. 2 kg is spent on the manufacture of one product of the first type, 4 kg of raw materials is spent on the manufacture of one product of the second type. Make a production plan that ensures the receipt of the greatest revenue from the sale of products, if it is necessary to produce no more than 40 products of the first type and no more than 20 products of the second type, and the selling price of one product of the first type is 3000 rubles. , and products of the second type-2000 rubles.

**Criteria for scoring various forms of current performance monitoring**

The criteria for a point assessment of various forms of current performance control are contained in the corresponding guidelines of the Department of Mathematics.

# Fund of assessment tools for carrying intermediate certification of students in the discipline.

The list of competencies with indicators of their achievement in the process of mastering the educational program is listed in paragraph **2** **"A list of planned results of the educational program, indicators of their achievement and planned results of training in the discipline."**

**Standard control tasks or other materials necessary for the assessment of indicators of competences, skills and knowledge**

|  |  |  |
| --- | --- | --- |
| **Competence code** | **Name of the competence** | **Tasks for assessing the formation of competence** |
| **PKN--2** | The ability to apply mathematical methods to solve standard professional problems, interpret the obtained mathematical results. | 1. In the web-like model, the demand function is , and the supply function – . The initial price is 3 d. e. Write out a general formula for the sequence of prices. Investigate the convergence of this sequence of prices. 2. Find the coefficients of the quadratic function  if the values at the specified points are given 3. The dependence of the product demand on its price is expressed by function . Find demand for goods, marginal demand and point elasticity of demand at a price, if *p* = 5. What is the average elasticity of demand , with an increase in the price by 4%? 4. The total values of three portfolios I, II, III of securities (stocks) of three different types A, B, C. The number of stocks in each portfolio and the values of the portfolios are indicated in the table:  |  |  |  |  |  | | --- | --- | --- | --- | --- | | Portfolio | Types of stock | | | Price | |  | A | B | C |  | | I  II  III | 9  20  15 | 5  3  6 | 7  5  4 | 80 000  82 000  78 000 |   Find the value of each stock.   1. The demand function for a certain product *D*(*p*) = 24 – 13*p* – 10*p*2 and the supply function for this product *S*(*p*) = 3*p*2 + 5*p* – 7 are given, where p is the price of the product in rubles. Calculate the elasticity of demand for the price at the point of market equilibrium.   6. The total costs for producing of *q* units are expressed by the function . The demand function for the product is , where p is the price of a unit of production. Find the minimum of average costs .  7. Worker's productivity per shift is described by a function , where *t* is the time in hours 0 ≤ *t* ≤ 8. Determine the output per day for a certain worker and the average productivity per hour.  8. Find the derivative of the function  at the point , if , , . |
| **UK-4** | Capability to use application software in solving professional tasks | 1. Determine whether the international trade of the two countries is balanced, if the national income vector of these countries is  and it's structural matrix .   1. The company produces two types of products, using the resources of three types. The technological matrix  and the vector of stocks are known . Draw a set of possible production plans. Make a production plan that ensures the highest revenue from the sale of products, if the price of the first type of product is 1000 rubles, and the second type of product is 2000 rubles. 2. In the three-branch balance model, the Leontiev matrix is given     and the vector of the norms of added value  for each industry  **а)** Find the equilibrium prices; **b)** Let there be an increase in the rate of added value of the first industry by 1.11. How many percent will the equilibrium prices of each industry increase? |

**Examples of practice-oriented (situational) tasks**

1. The demand function for some goods *D*(*p*) = 120 – 19*p* – 20*p*2 and the supply function of this product *S*(*p*) = 3*p*2 + 4*p* – 18 are given, where *p* the price of goods in rubles. Calculate the price elasticity of demand at the point of market equilibrium.
2. The company produces 3 types of products in quantities characterized by the vector  For its production 5 types of raw materials are used. Raw material costs (in kg per unit of production) are characterized by the matrix



if  – the consumption of the *i*-th type of raw materials per unit of the *j-*th type of products, the vector  sets the unit cost of each type of raw material.

Determine: a) the amount of raw materials of each type necessary to ensure the plan; b) the total cost of raw materials necessary for the production of all products.

1. Worker's productivity per shift is described by the function , where *t* is the time in hours, 0 ≤ *t* ≤ 8. Determine the output per day for a given worker and the average productivity per hour. Determine the volume of production in 5 working days by a team consisting of 7 people.
2. Find the derivative of the function  at the point , if , , .

**Examples of test tasks.**

1. Find the antiderivative  of the function 



2. Find the inverse of the matrix :



3. Find the image of the vector  if  is a linear operator and 

## 

1. Using the Lagrange method, transform the quadratic form  to a normal one:

а) ,

б) **,**

в) **,**

г) **,**

The assessment for the discipline can be set as a result of the average score for all the competencies formed by the discipline.

**Questions for the exam (1 semester).**

1. Sets. Operations on sets, finite, countable and uncountable sets. Bounded and unbounded sets.
2. Complex numbers and a operations on them. Algebraic and trigonometric forms of notation of complex numbers.
3. The concept of a function. Properties of functions of a single variable.
4. Functions in economics.
5. Numerical sequences, limit of a sequence and its properties, monotone, limited sequences.
6. Simple and compound interest. Accumulation and discounting. Continuous accrual of interest.
7. A spider-shaped model of the single-product market.
8. Number row. The convergence of the series. The amount of the row.
9. The limit of a function at a point and on infinity.
10. Infinitely small and infinitely large functions.
11. The first and the second remarkable limits.
12. Comparison of infinitely large and infinitely small functions.
13. Continuity of a function at a point and on a set. Properties of continuous functions.
14. Points of discontinuity and their classification.
15. Asymptotes of the graph of a function.
16. Derivative of a function, its geometric meaning, the properties of the derivative.
17. Derivative of composite and implicitly defined functions.
18. Marginal and average costs in the economy (for a single variable function).
19. Elasticity of the function (for a single variable function).
20. Differentiability of a function, the first differential and its geometric meaning.
21. Fundamental theorems of differential calculus: Rolle's and Lagrange's theorems.
22. The L'Hôpital's rule.
23. Monotonicity of a function. Monotonicity condition.
24. Extremum of a function. Necessary and sufficient conditions for an extremum.
25. The largest and smallest value of the function on the segment.

26. Higher-order derivatives and differentials.

27. Taylor's formula. Maclaurin's formula.

28.Expansion of elementary functions according to the Maclaurin formula.

29. Convexity and concavity of the graph of the function . Inflection points.

30. Antiderivative of a function and its properties.

31. Basic methods of integration: change of variable, integration by parts.

32. Definite integral. Newton-Leibniz formula.

33. The average value of a function.

34. Improper integrals. Poisson's integral..

35. The space *Rn*. Set in *Rn*. Functions of several variables.

36. Examples of the functions of several variables in the economy.

37. The limit and continuity of the function of several variables.

38. Partial derivatives of the function of several variables.

39. Differentiability and differential function of several variables.

40. Limit and averages in the economy (case of several variables).

41. Elasticity of the function (case of several variables function).

42. Derivative of a composite function.

43. The directional derivative and the gradient.

**Questions for the exam (2 semester).**

1. Local extremum of a function of several variables. Necessary conditions for a local extremum.

2. A sufficient condition for the case of two independent variables.

3. Conditional extremum. Substitution method.

4. Conditional extremum. Lagrange multiplier method.

5. Global extremum.

6. Multiple integrals. Reduction of a multiple integral to a repeated one.

7. General solution of a differential equation. Particular solutions of a differential equation. Cauchy problem.

8. Separable differential equations.

9. First order homogeneous equations.

10. Linear equation of the first order.

11. Bernoulli's equation.

12. Linear differential equations with constant coefficients.

13. Sustainability of the solution. Stability criterion.

14. Arithmetic vectors.

15. Matrices and their types. Linear operations on matrices. Matrix transposition. Product of matrices.

16. Elementary transformations over rows and columns of matrices.

17. A theorem on reducing an arbitrary matrix to a row-echelon form and reduced row-echelon form. The rank of the matrix. Non-degeneracy of square matrices.

18. Inverse matrix.

19. Determinant of a square matrix. Decomposition of the determinant by row or column. Determinant properties. Criterion for non-degeneracy of a matrix.

20. System of linear algebraic equations (SLAE). Kronecker-Capelli theorem.

21. Straight lines on a plane.

22. Straight lines and planes in space.

23. Systems of linear algebraic inequalities and their applications in economics.

24. Linear (vector) space.

25. Linear dependence (independence) of the vector system. Basis and dimension of linear space.

26. Linear transformations of the space Rn (linear operators).

27. Eigenvalues and Eigenvectors of the matrix.

28. Linear exchange model (international trade model).

29. Symmetric matrices and quadratic forms.

30. Reduction of a quadratic form to normal and canonical forms.

31. Second-order curves.

32. Examples of linear optimization models in economics.

33. Formulation and various forms of the linear programming problem. Geometric interpretation of a linear programming problem.

34. The canonical form of the linear programming problem. Feasible solutions. Properties of the feasible set.

35. Algorithm of the simplex method for linear programming.

36. Simplex method as a method of directed enumeration of basic feasible solutions. Optimality criterion.

37. Symmetrical pair of dual problems. Economic interpretation of the dual problem.

38. The main inequality of the theory of duality, its economic interpretation.

39. Small duality theorem.

40. A sufficient condition for the optimality of a pair of mutually dual problems.

41. The first and second main theorems of duality, their geometric and economic interpretation.

42. An asymmetric pair of dual problems.

43. The third main theorem of duality, its geometric and economic interpretation.

44. Transportation problem.

45. A task that is dual to the transport one.

46. A closed transportation problem and its solution by the method of potentials. Economic interpretation of cell estimates, supplier and consumer potentials.

47. Degenerate transportation problem.

**Example of a test ticket (1 semester)**

1. Derivative of a composite and implicitly defined functions.
2. Calculate the limit of function .

3. The demand function for a good D(p) = 82 – 15p – 10p2 and the supply function for this good S(p) = 2p2 + 3p – 2 are given, where p is the price of the product in rubles. Calculate the price elasticity of demand at the point of market equilibrium.

4. Investigate the convexity of the function 

5. The labor productivity of one worker per shift is defined by the function , where t is the time in hours, 0 ≤ t ≤ 8. Determine the volume of production output for 5 working days by a team consisting of 8 people.

6. Find the derivative of the function  at a point , if , , .

**Example of an exam ticket (2 semester)**

1. Matrices and their types. Linear operations on matrices. Matrix transposition. Product of matrices.

2. Find the extremum of the function 

3. Solve the differential equation 

4. Explore the system of vectors for a linear dependence

.

5. Solve a system of linear algebraic equations



6. The company produces two types of products, using three types of resources. The technological matrix  the vector of reserves  are known.

Draw a set of possible production plans. Make a production plan that ensures the receipt of the greatest revenue from the sale of products, if the price of the first good is 2000 rubles and the price of the second good – 3000 rubles.

**7. Mandatory and optional reading list**

1. Высшая математика для экономического бакалавриата: учебник и практикум / Н.Ш. Кремер [и др.]; под ред. Н.Ш. Кремера. – 5-е изд.; перераб. и доп. – М.: Юрайт, 2021. – ЭБС Юрайт.
2. Математика для экономистов и менеджеров [электронный ресурс]: Учебник / под ред. Н.Ш. Кремера. – М.: Кнорус, 2021. ЭБС: book.ru.
3. Математика для экономистов и менеджеров [электронный ресурс]: Практикум: учебное пособие / Н.Ш. Кремер, Б.А. Путко, М.Н. Фридман / под ред. Н.Ш. Кремера. – М.: Кнорус, 2015. ЭБС: book.ru.
4. Линейная алгебра: учебник и практикум / Н.Ш. Кремер, М.Н.Фридман, И.М.Тришин / под ред. Н.Ш. Кремера. – 3-е изд., перераб. и доп. – М.: Юрайт, 2021. – ЭБС Юрайт.
5. Математика в экономике. Ч.1, Ч.2: учебник / А.С. Солодовников [и др.]. – 3-е изд., перераб. и доп. – М.: Финансы и статистика; Инфра-М, 2011.
6. Сборник задач по курсу "Математика в экономике". В 3 ч. : учебное пособие / под ред. В.А. Бабайцева и В.Б. Гисина. – М.: Финансы и статистика, 2013.
7. Красс М.С. Математика для экономических специальностей: учебник / М.С. Красс. – 4. изд., испр. – М.: Дело, 2005, 2002.
8. Кремер Н.Ш. Математика для экономистов: от арифметики до эконометрики: учебно-справочное пособие / Кремер Н.Ш., Путко Б.А., И.М. Тришин.; Финуниверситет; под ред. Н.Ш. Кремера. – 4-е изд., перераб. и доп. – М.: Юрайт, 2019. – 724с. – ЭБС Юрайт.
9. Simon C.P. Mathematics for Economists / C.P. Simon, L. Blume. – Norton Company. N.-Y., 1994.
10. Dowling E. Mathematical Economics / E. Dowling. – Shaum’s Outline Series. N.-Y., 2006.

**8. List of IT resources, incl. the list of software, information and reference systems (as appropriate).**

**8. 1. Software:**

1. Windows, Microsoft Office software;

2. ESET Endpoint Security antivirus software;

3. Information and educational portal of the Financial University under the Government of the Russian Federation [http://portal.](http://portal.ufrf.ru/) [ufrf.](http://portal.ufrf.ru/) [ru/](http://portal.ufrf.ru/).

4. The website of the Department of Data Analysis, Decision-Making and Financial Technology. [http://fa.](http://fa.ru/dep/data_analysis/) [ru/dep/data\_analysis/](http://fa.ru/dep/data_analysis/)

**8.2. Databases and information and reference systems**

1. Garant information and reference system;

2. Consultant Plus legal information system;

3. <http://ru.wikipedia.org/wiki/Wiki> e-encyclopedia;

4. <http://www.skrin.ru/>. database; etc.

5.The Electronic Library of the Financial University (EB) <http://elib.fa.ru/>

6. The electronic library system of the publishing house «ЮРАЙТ» <https://www.urait.ru/>

7. National Electronic Library <http://нэб.рф/>

8. Mass open online-course/specialization "Remember everything! School Mathematics for Freshmen" / Financial University under the Government of Russian Federation – <https://online.fa.ru/courses/course-v1:fa+adaptmathem+2021/about>

1. To be filled in when the updated Financial University educational standards and federal state educational standards of higher education “3++” are implemented. [↑](#footnote-ref-2)
2. Financial University under the Government of the Russian Federation

   Department of Mathematics

   Galina A. Postovalova

   **Mathematics**

   **SYLLABUS**

   ***Level of Study:*** *Bachelor’s Degree*

   ***Fields of Study:*** *Management*

   ***Study Programs:***

   *Management, Business Management, Bachelor Business Administration* [↑](#endnote-ref-2)